Graduate Journal of Interdisciplinary Research, Reports & Reviews
ark:/85846/JPR3.2024AR.0102049
ISSN(E): 2584-2919

Grad.J.InteR^3, 2024, Vol.01, No. 02
pp.49-56

Research Article

On Vertex-Based Dimension of Some Graphs Joining Certain Prism Graphs

M. Gayathri

Department of Mathematics, Government First Grade College, Varthur, Bangalore-560087
Affiliated to Bangalore North University.
E-mail: *gayathrimanikonda@gmail.com;*
ORCID:0009-0002-2908-9451

(Received:17-03-24; Accepted: 03- 04-24)

ABSTRACT:
Background: In graph theory, the prism graph is a type of graph that is characterised by having the structure of a prism as its underlying framework. The notion of a resolving set and that of metric dimension for a graph of a prism is important in uniquely identifying the vertices within a prism graph. For a non-trivial connected graph $\Gamma_r = \Gamma_r(V, E)$, an ordered subset U of vertices $resolves$ any pair of different vertices $y_1, y_2 \in V$, if $d(v, y_1) \neq d(v, y_2)$ for some $v \in U$. Such a set U is said to be a resolving set for Γ_r and the smallest cardinality of U is called the $metric\ dimension$ of Γ_r.
Purpose: The purpose of this article is to determine the notion of resolving sets and their corresponding metric dimensions for two complex families of planar graphs obtained by joining $m-$copies of the prism graph on known families of convex polytope graphs.
Methods: The methods used are purely theoretical, based on mathematical reasoning and established definitions related to graph theory.
Results: In this article, we have determined successfully the resolving set and metric dimension for two specific complex families of planar graphs, denoted by L_n and M_n, constructed using $m-$copies of a prism graph. These findings contribute to our understanding of these concepts within graph theory.
Conclusions: This research indicates the importance of studying resolving sets and metric dimensions in various graph structures, particularly those derived from multiple copies of the prism graph connected through known families of convex polytope graphs. This work may inspire further investigations into similar graph families or other applications of these concepts in different areas of mathematics and computer science.
Keywords: Metric dimension, independent set, basis set, convex polytope, prism graphs
2000 Mathematics Subject Classification: 05C12

1. Introduction

It has been observed that the graph of an k-gonal (where $k \in \mathbb{N}$ and $k \geq 3$) prism exhibits a distinct pattern in terms of its vertices and edges. Specifically, it has been determined that an k-gonal prism possesses $2k$ vertices and $3k$ edges. This finding provides valuable insight into the structural properties of k-gonal prisms and contributes to our understanding of their geometric characteristics. The graphs in question exhibit regularity and possess a cubic structure. Prism graphs possess the property of vertex-transitivity due to the presence of symmetries that map each vertex to every other vertex. As polyhedral graphs, they exhibit the property of being 3-vertex-connected planar graphs. It has been observed that every prism graph, which is a specific type of graph formed by connecting two copies of a cycle graph with corresponding vertices, possesses a Hamiltonian cycle. A Hamiltonian cycle is a cycle that visits each vertex exactly once. Researchers have extensively studied this property and proven its validity for all prism graphs [1].

Throughout this article, all graphs under consideration are connected, non-trivial, undirected, and simple. Let $\Gamma_r = (V, E)$ be a graph with $V(\Gamma_r)$ and $E(\Gamma_r)$ as its vertex and edge set respectively. The shortest length path between two distinct vertices y_1 and y_2 in $V(\Gamma_r)$, is referred to as the distance $(d(y_1, y_2))$ between y_1 and y_2 in Γ_r. The number of distinct edges incident on a vertex y in Γ_r is known as the degree of v (denoted by d_y). Two vertices y_1 and y_2 in Γ_r are said to resolved by a vertex y, if $d(y, y_1) \neq d(y, y_2)$ in Γ_r. Then, a subset $U \subseteq V(\Gamma_r)$ with this property, i.e., every pair of unequal vertices in Γ_r can be resolved by at least one member of U, is said to be a resolving set (RS) for Γ_r. The smallest cardinality set U with resolving characteristic is called the metric basis (MB) for Γ_r, and the MB set cardinality is the metric dimension (MD) for Γ_r, represented by $dim(\Gamma_r)$ [2, 3].

For a subset $U = \{y_1, y_2, y_3, ..., y_s\}$ of distinct ordered vertices in $V(\Gamma_r)$, the unique s-length tuple code for each $q \in V(\Gamma_r)$ is given as follows $\zeta(q|U) = (d(q, y_1), d(q, y_2), d(q, y_3), ..., d(q, y_s))$. Using this fact, the subset U is a RS for Γ_r, if $\zeta(y_1|U) \neq \zeta(y_2|U)$, for every pair of different vertices $y_1, y_2 \in V(\Gamma_r)$. Next, a subset U in $V(\Gamma_r)$ with distinct vertices is said to be a resolving independent set for Γ_r, if it is (i) independent set as well as (ii) a RS in Γ_r. A proper subset of a RS is not necessarily a RS, while a superset of every RS is always a RS [4].

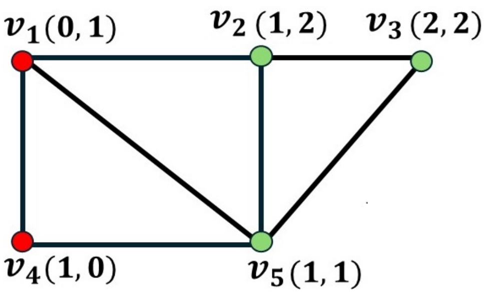

Figure 1: Graph Γ^*

To understand the concept of RS and MD, let us consider a graph Γ^* on 5 vertices and 7 edges, as shown in Fig. 1. To find the metric dimension of Γ^*, we suppose that $U_1 = \{v_1, v_4\}$ (red color vertices in Γ^*). Next, metric codes for each vertex in Γ^* with respect to U_1 are as follows: $\zeta(v_1|U_1) = (0, 1)$, $\zeta(v_2|U_1) = (1, 2)$, $\zeta(v_3|U_1) = (2, 2)$, $\zeta(v_4|U_1) = (1, 0)$, and $\zeta(v_5|U_1) = (1, 1)$. From this, we find that the metric codes for all the vertices in Γ^* corresponding to the set U_1 are unique, and so we say that U_1 is a resolving set for Γ^*. Also, U_1 is the minimum resolving set for Γ^*, as the cardinality of U_1 is 2 [5]. Hence, we have concluded that $dim(\Gamma^*) = 2$.

Slater [3] and Harary & Melter [2] independently introduced the concept of MD of a graph in 1970s. There is a wide range of literature available on MD that addresses both theoretical and practical aspects. The MD has appeared in various areas including combinatorial optimization, sonar, pharmaceutical chemistry, robot navigation, graph isomorphism testing, and many more see [6, 7, 8, 9, 10] and references therein.

The computation of MD for distinct graph families, is always a challenging task because deciding and selecting of a landmark (resolving) vertices in minimum numbers is not that easy due to the complexity and scalability of the considered graph network. Further, the notion of MD was extended as well as investigated by eminent researchers from time to time and named them as the variants of MD [11]. Several authors have studied MD and its related variants for several distinct graphs as well as for various other graph-theoretic aspects, for instance prism graph, path graph, complete graph, cycle graph, cycle graph with chords, antiprism graph, several ladder graphs (pentagonal, heptagonal, etc), convex polytope graph, wheel graph, tadpole graph, kayak paddle graph, and numerous planar and chemical graphs [4, 6, 7, 9, 10, 12, 13]. Even though after investigating these notions for the large number of graph families, there are still many families for which these notions are not investigated to date.
In this paper, two planar graph families, viz., L_n and M_n has been constructed, which are obtained by taking m-copies $(q = m)$ of the prism graph on the two known convex polytope graphs R_n [9] and U_n [7], respectively. For these so obtained graphs, we investigate their minimal MB sets and finally obtain their MD. To carry out these results, we need the following result:

Proposition 1.1. *[5] For a connected graph Γ_r with metric dimension two i.e., the metric basis U for Γ_r has cardinality two, and say $U = \{y_1, y_2\}$. Then, the following three points for Γ_r are true:*

1. *Always shortest unique path P between y_1 and y_2 exist,*
2. *d_{y_1} and d_{y_2} is at most 3, and*
3. *The d_v is at most 5, for any $v \in P$.*

The following paper is structured as follows: In Section 2, we consider an infinite family of convex polytope L_n and find its minimum RS with its respective MD. In Section 3, we consider an infinite family of convex polytope M_n and find its minimum RS with its respective MD. Finally, the conclusion and future scope of the manuscript is presented.

2. Minimum Vertex Resolving Number of L_n

In this section, we investigate some of the basic properties and the MD of a planar graph L_n. The graph L_n consists of $n(q+3)$ vertices and $n(2q+5)$ edges (see Fig.2). The sets containing vertices and edges for planar graph L_n are denoted by $V(L_n)$ and $E(L_n)$ respectively, where $E(L_n) = \{j_\alpha j_{\alpha+1}, j_\alpha k_\alpha, k_\alpha j_{\alpha+1}, k_\alpha l_\alpha, l_\alpha m_\alpha^1, m_\alpha^1 l_{\alpha+1}, m_\alpha^s m_{\alpha+1}^s : 1 \le \alpha \le n, 1 \le s \le q\} \cup \{m_\alpha^s m_\alpha^{s+1} : 1 \le \alpha \le n, 1 \le s \le q-1\}$ and $V(L_n) = \{j_\alpha, k_\alpha, l_\alpha, m_\alpha^s : 1 \le \alpha \le n, 1 \le s \le q\}$.

We call vertices $\{j_\alpha : 1 \le \alpha \le n\}$, as j-cycle vertices in L_n, the vertices $\{k_\alpha, l_\alpha : 1 \le \alpha \le n\}$, as inner vertices in L_n, and the vertices $\{m_\alpha^s : 1 \le \alpha \le n, 1 \le s \le q\}$, as outer vertices in L_n. In the following result, we investigate the MD of L_n.

Theorem 2.1. $dim(L_n) = 3$, *where $n \ge 6$ is a positive integer.*

Proof. Now, the following cases, which depend on the natural n, can be employed to investigate this result.

Case(I) $n \equiv 0(mod\ 2)$.
We set $n = 2y$; $y \in \mathbb{Z}^+$ and $y \ge 3$. Let $U = \{j_2, j_{y+1}, j_n\} \subset V(L_n)$. Next, each vertex of L_n has given metric coordinate corresponding to the taken set U.
For vertices over j-cycle, i.e., $\{j_\alpha : 1 \le \alpha \le n\}$, the metric co-ordinates are

$$\zeta(j_\alpha|U) = \begin{cases} (1, y, \alpha), & \alpha = 1; \\ (\alpha-2, y-\alpha+1, \alpha) & 2 \le \alpha \le y; \\ (\alpha-2, y-\alpha+1, 2y-\alpha) & \alpha = y+1; \\ (2y-\alpha+2, \alpha-y-1, 2y-\alpha) & y+2 \le \alpha \le 2y. \end{cases}$$

For the vertices $\{k_\alpha : 1 \le \alpha \le n\}$, the metric co-ordinates are

$$\zeta(k_\alpha|U) = \begin{cases} (1, y-\alpha+1, \alpha+1) & \alpha = 1; \\ (\alpha-1, y-\alpha+1, \alpha+1) & 2 \le \alpha \le y-1; \\ (\alpha-1, y-\alpha+1, 2y-\alpha) & \alpha = y; \\ (\alpha-1, \alpha-y, 2y-\alpha) & \alpha = y+1; \\ (2y-\alpha+2, \alpha-y, 2y-\alpha) & y+2 \le \alpha \le 2y-1; \\ (2y-\alpha+2, \alpha-y, 1) & \alpha = 2y. \end{cases}$$

For the vertices $\{l_\alpha : 1 \le \alpha \le n\}$, the metric co-ordinates are $\zeta(l_\alpha|U) = \zeta(k_\alpha|U) + (1,1,1)$ for $1 \le \alpha \le n$. Next, for the vertices $\{m_\alpha^1 : 1 \le \alpha \le n\}$, the metric co-ordinates are

$$\zeta(m_\alpha^1|U) = \begin{cases} (3, y-\alpha+2, \alpha+3) & \alpha = 1; \\ (\alpha+1, y-\alpha+2, \alpha+3) & 2 \le \alpha \le y-1; \\ (\alpha+1, 3, 2y-\alpha+1) & \alpha = y; \\ (2y-\alpha+3, \alpha-y+2, 2y-\alpha+1) & y+1 \le \alpha \le 2y-2; \\ (2y-\alpha+3, \alpha-y+2, 3) & 2y-1 \le \alpha \le 2y. \end{cases}$$

Finally, for the vertices $\{m_\alpha^s : 1 \le \alpha \le n, 2 \le s \le q\}$, the co-ordinates are $\zeta(m_\alpha^s|U) = \zeta(m_\alpha^1|U) + (s-1, s-1, s-1)$ for $1 \le \alpha \le n$. Next, these codes for all vertices in L_n are unique and distinct from one and an other in at least one co-ordinate, which results in $dim(L_n) \le 3$.

Now, for reverse inequality i.e., $dim(L_n) \ge 3$, we show that no set U with $|U| = 2$, form a RS for L_n. Assuming $dim(L_n) = 2$ (on contrary). Using proposition 1, we have following to discuss for RS U with $|U| = 2$ in L_n:

1. Let $U = \{k_1, k_g\}$, k_g $(2 \le g \le y+1)$, then $\zeta(j_n|U) = \zeta(k_n|U)$, for $2 \le g \le y-1$, $\zeta(l_2|U) = \zeta(k_{n-1}|U)$, when $g = y$, and

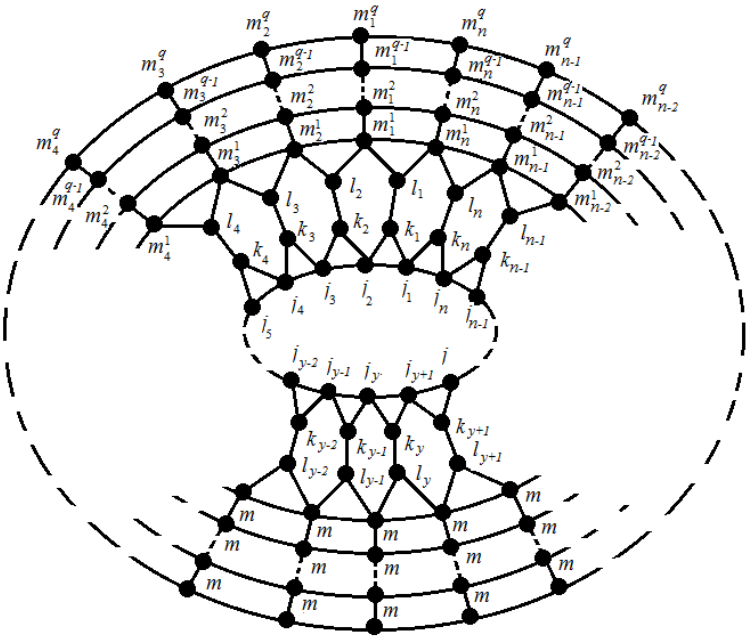

Figure 2: The Graph L_n

$\zeta(j_2|U) = \zeta(j_1|U)$, when $g = y+1$, a contradiction.

2. Let $U = \{l_1, l_g\}$, l_g $(2 \leq g \leq y+1)$, then $\zeta(j_n|U) = \zeta(k_n|U)$, for $2 \leq g \leq y-1$, $\zeta(p_3|U) = \zeta(m_2^2|U)$, when $g = y$, and $\zeta(j_2|U) = \zeta(j_1|U)$, when $g = y+1$, a contradiction.

3. Let $U = \{m_1^q, m_g^q\}$, m_g^q $(2 \leq g \leq y+1)$, then $\zeta(m_1^{q-1}|U) = \zeta(m_n^q|U)$, for $2 \leq g \leq y$, and $\zeta(m_2^q|U) = \zeta(m_n^q|U)$, when $g = y+1$, a contradiction.

4. Let $U = \{k_1, l_g\}$, l_g $(1 \leq g \leq y+1)$, then $\zeta(j_n|U) = \zeta(k_n|U)$, for $1 \leq g \leq y-1$, $\zeta(m_1^1|U) = \zeta(p_3|U)$, when $g = y$, and $\zeta(j_2|U) = \zeta(j_1|U)$, when $g = y+1$, a contradiction.

5. Let $U = \{k_1, m_g^q\}$, m_g^q $(1 \leq g \leq y+1)$, then $\zeta(j_1|U) = \zeta(j_2|U)$, for $g = 1$, $\zeta(m_n^1|U) = \zeta(k_2|U)$, when $2 \leq g \leq y$, and $\zeta(m_{y+1}^1|U) = \zeta(l_{y+2}|U)$, when $g = y+1$, a contradiction.

6. Let $U = \{l_1, m_g^q\}$, m_g^q $(1 \leq g \leq y+1)$, then $\zeta(j_1|U) = \zeta(j_2|U)$, for $g = 1$, $\zeta(m_{n-1}^1|U) = \zeta(j_2|U)$, when $2 \leq g \leq y-1$, $\zeta(m_y^1|U) = \zeta(m_{y-1}^2|U)$, when $g = y$, and $\zeta(m_{y+1}^1|U) = \zeta(m_{y+2}^2|U)$, when $g = y+1$, a contradiction.

Thus, from this we have $dim(L_n) \geq 3$, implying that $dim(L_n) = 3, \forall\, n \geq 6$.

Case(II) $n \equiv 1(mod\ 2)$.

We set $n = 2y+1$; $y \in \mathbb{Z}^+$ and $y \geq 3$. Let $U = \{j_2, j_{y+1}, j_n\} \subset V(L_n)$. Next, each vertex of L_n has given metric coordinate corresponding to the taken set U.

For vertices over j-cycle, i.e., $\{j_\alpha : 1 \leq \alpha \leq n\}$, the metric co-ordinates are

$$\zeta(j_\alpha|U) = \begin{cases} (1, y, \alpha) & \alpha = 1; \\ (\alpha-2, y-\alpha+1, \alpha) & 2 \leq \alpha \leq y; \\ (\alpha-2, y-\alpha+1, 2y-\alpha+1) & \alpha = y+1; \\ (\alpha-2, \alpha-y-1, 2y-\alpha+1) & \alpha = y+2; \\ (2y-\alpha+3, \alpha-y-1, 2y-\alpha+1) & y+3 \leq \alpha \leq 2y+1. \end{cases}$$

For the vertices $\{k_\alpha : 1 \leq \alpha \leq n\}$, the metric co-ordinates are

$$\zeta(k_\alpha|U) = \begin{cases} (1, y-\alpha+1, \alpha+1) & \alpha = 1; \\ (\alpha-1, y-\alpha+1, \alpha+1) & 2 \leq \alpha \leq y; \\ (\alpha-1, \alpha-y, 2y-\alpha+1) & \alpha = y+1; \\ (2y-\alpha+3, \alpha-y, 2y-\alpha+1) & y+2 \leq \alpha \leq 2y; \\ (2y-\alpha+3, \alpha-y, 1) & \alpha = 2y+1. \end{cases}$$

For the vertices $\{l_\alpha : 1 \leq \alpha \leq n\}$, the metric co-ordinates are $\zeta(l_\alpha|U) = \zeta(k_\alpha|U) + (1, 1, 1)$ for $1 \leq$

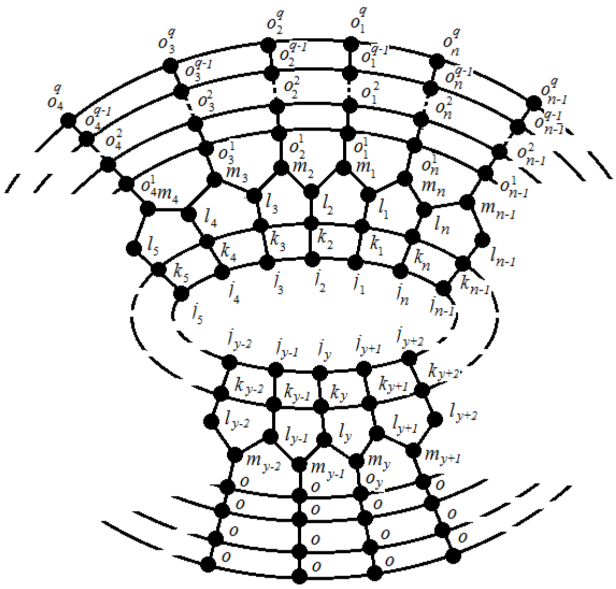

Figure 3: The Graph M_n

$\alpha \leq n$. Next, for the vertices $\{m^1_\alpha : 1 \leq \alpha \leq n\}$, the co-ordinates are

$$\zeta(m^1_\alpha|U) = \begin{cases} (3, y-\alpha+2, \alpha+3) & \alpha = 1; \\ (\alpha+1, y-\alpha+2, \alpha+3) & 2 \leq \alpha \leq y-1; \\ (\alpha+1, 3, 2y-\alpha+2) & \alpha = y; \\ (2y-\alpha+4, \alpha-y+2, 2y-\alpha+2) & y+2 \leq \alpha \leq 2y-1; \\ (2y-\alpha+4, \alpha-y+2, 3) & \alpha = 2y; \\ (2y-\alpha+4, y+2, 3) & \alpha = 2y+1. \end{cases}$$

Finally, for the vertices $\{m^s_\alpha : 1 \leq \alpha \leq n, 2 \leq s \leq q\}$, the metric co-ordinates are $\zeta(m^s_\alpha|U) = \zeta(m^1_\alpha|U) + (s-1, s-1, s-1)$ for $1 \leq \alpha \leq n$. Next, these codes for all vertices in L_n are unique and distinct from one and an other in at least one co-ordinate, which results in $dim(\mathrm{L}_n) \leq 3$. Assuming that $dim(\mathrm{L}_n) = 2$, then as in Case (I), we have the same contradictions. Therefore, we have $dim(\mathrm{L}_n) = 3$ as well in this case, which proofs the theorem. □

Proposition 2.2. *The resolving independent number for* L_n *is* 3, $\forall\, n \geq 6$.

Proof. For the proof, follow Theorem 2.1. □

3. Minimum Vertex Resolving Number of M_n

In this section, we investigate some of the basic properties and the MD of a planar graph M_n (see Fig. 3). The graph M_n consists of $n(q+4)$ vertices and $n(2q+6)$ edges. The sets containing vertices and edges for planar graph M_n are denoted by $V(\mathrm{M}_n)$ and $E(\mathrm{M}_n)$ respectively, where $E(\mathrm{M}_n) = \{j_\alpha j_{\alpha+1}, j_\alpha k_\alpha, k_\alpha k_{\alpha+1}, k_\alpha l_\alpha, l_\alpha m_\alpha, m_\alpha l_{\alpha+1}, m_\alpha o^1_\alpha, o^s_\alpha o^s_{\alpha+1} : 1 \leq \alpha \leq n, 1 \leq s \leq q\} \cup \{o^s_\alpha o^{s+1}_\alpha : 1 \leq \alpha \leq n, 1 \leq s \leq q-1\}$ and $V(\mathrm{M}_n) = \{j_\alpha, k_\alpha, l_\alpha, m_\alpha, o^s_\alpha : 1 \leq \alpha \leq n, 1 \leq s \leq q\}$.

We call vertices $\{j_\alpha : 1 \leq \alpha \leq n\}$, as j-cycle vertices in M_n, the vertices $\{k_\alpha : 1 \leq \alpha \leq n\}$, as k-cycle vertices in M_n, the vertices $\{l_\alpha, m_\alpha : 1 \leq \alpha \leq n\}$, as lm-cycle vertices in M_n, and the vertices $\{o^s_\alpha : 1 \leq \alpha \leq n, 1 \leq s \leq q\}$ as outer vertices in M_n. In the following result, we investigate the MD of M_n.

Theorem 3.1. $dim(\mathrm{M}_n) = 3$*, where* $n \geq 6$ *is a positive integer.*

Proof. Now, the following cases, which depend on the natural n, can be employed to investigate this result.

Case(I) $n \equiv 0(mod\ 2)$.
We set $n = 2y$; $y \in \mathbb{Z}^+$ and $y \geq 3$. Let $U = \{j_2, j_{y+1}, j_n\} \subset V(\mathrm{M}_n)$. Next, each vertex of M_n has given metric coordinate corresponding to the taken set U.
For vertices over j-cycle, i.e., $\{j_\alpha : 1 \leq \alpha \leq n\}$, the metric co-ordinates are

$$\zeta(j_\alpha|U) = \begin{cases} (1, y, \alpha) & \alpha = 1; \\ (\alpha-2, y-\alpha+1, \alpha) & 2 \leq \alpha \leq y; \\ (\alpha-2, y-\alpha+1, 2y-\alpha) & \alpha = y+1; \\ (2y-\alpha+2, \alpha-y-1, 2y-\alpha) & y+2 \leq \alpha \leq 2y. \end{cases}$$

For the vertices $\{k_\alpha : 1 \le \alpha \le n\}$, the metric co-ordinates are $\zeta(k_\alpha|U) = \zeta(j_\alpha|U) + (1,1,1)$ for $1 \le \alpha \le n$. Next, for the vertices $\{l_\alpha : 1 \le \alpha \le n\}$, the metric co-ordinates are $\zeta(l_\alpha|U) = \zeta(k_\alpha|U) + (1,1,1)$ for $1 \le \alpha \le n$.
For the vertices $\{m_\alpha : 1 \le \alpha \le n\}$, the metric co-ordinates are

$$\zeta(m_\alpha|U) = \begin{cases} (3, y-\alpha+3, \alpha+3) & \alpha = 1; \\ (\alpha+1, y-\alpha+3, \alpha+3) & 2 \le \alpha \le y-1; \\ (\alpha+1, y-\alpha+3, 2y-\alpha+2) & \alpha = y; \\ (\alpha+1, \alpha-y+2, 2y-\alpha+2) & \alpha = y+1; \\ (2y-\alpha+4, \alpha-y+2, 2y-\alpha+2) & y+2 \le \alpha \\ & \le 2y-1; \\ (2y-\alpha+4, \alpha-y+2, 3) & \alpha = 2y. \end{cases}$$

Finally, for the vertices $\{o_\alpha^s : 1 \le \alpha \le n, 1 \le s \le q\}$, the metric co-ordinates are $\zeta(o_\alpha^s|U) = \zeta(m_\alpha|U) + (s,s,s)$ for $1 \le \alpha \le n$.

Next, these codes for all vertices in M_n are unique and distinct from one and an other in at least one co-ordinate, which results in $dim(M_n) \le 3$. Now, for reverse inequality i.e., $dim(M_n) \ge 3$, we show that no set U with $|U| = 2$, form a RS for M_n. Now, for reverse inequality i.e., $dim(M_n) \ge 3$, we show that no set U with $|U| = 2$, form a RS for M_n. Assuming $dim(M_n) = 2$ (on contrary). Using proposition 1, we have following to discuss for RS U with $|U| = 2$ in M_n:

1. Let $U = \{k_1, k_g\}$, k_g $(2 \le g \le y+1)$, then $\zeta(j_n|\mathbb{R}) = \zeta(k_n|\mathbb{R})$, for $2 \le g \le y-1$; $\zeta(l_2|\mathbb{R}) = \zeta(k_{n-1}|\mathbb{R})$, when $g = y$; and $\zeta(j_2|\mathbb{R}) = \zeta(j_1|\mathbb{R})$, when $g = y+1$, a contradiction.

2. Let $U = \{l_1, l_g\}$, l_g $(2 \le g \le y+1)$, then $\zeta(j_n|\mathbb{R}) = \zeta(k_n|\mathbb{R})$, for $2 \le g \le y-1$; $\zeta(p_3|\mathbb{R}) = \zeta(m_2^2|\mathbb{R})$, when $g = y$; and $\zeta(j_2|\mathbb{R}) = \zeta(j_1|\mathbb{R})$, when $g = y+1$, a contradiction.

3. Let $U = \{m_1^q, m_g^q\}$, m_g^q $(2 \le g \le y+1)$, then $\zeta(m_1^{q-1}|\mathbb{R}) = \zeta(m_n^q|\mathbb{R})$, for $2 \le g \le y$; and $\zeta(m_2^q|\mathbb{R}) = \zeta(m_n^q|\mathbb{R})$, when $g = y+1$, a contradiction.

4. Let $U = \{k_1, l_g\}$, l_g $(1 \le g \le y+1)$, then $\zeta(j_n|\mathbb{R}) = \zeta(k_n|\mathbb{R})$, for $1 \le g \le y-1$; $\zeta(m_1^1|\mathbb{R}) = \zeta(p_3|\mathbb{R})$, when $g = y$; and $\zeta(j_2|\mathbb{R}) = \zeta(j_1|\mathbb{R})$, when $g = y+1$, a contradiction.

5. Let $U = \{k_1, m_g^q\}$, m_g^q $(1 \le g \le y+1)$, then $\zeta(j_1|\mathbb{R}) = \zeta(j_2|\mathbb{R})$, for $g = 1$; $\zeta(m_n^1|\mathbb{R}) = \zeta(k_2|\mathbb{R})$, when $2 \le g \le y$; and $\zeta(m_{y+1}^1|\mathbb{R}) = \zeta(l_{y+2}|\mathbb{R})$, when $g = y+1$, a contradiction.

6. Let $U = \mathbb{R} = \{l_1, m_g^q\}$, m_g^q $(1 \le g \le y+1)$, then $\zeta(j_1|\mathbb{R}) = \zeta(j_2|\mathbb{R})$, for $g = 1$; $\zeta(m_{n-1}^1|\mathbb{R}) = \zeta(j_2|\mathbb{R})$, when $2 \le g \le y-1$; $\zeta(m_y^1|\mathbb{R}) = \zeta(m_{y-1}^2|\mathbb{R})$, when $g = y$; and $\zeta(m_{y+1}^1|\mathbb{R}) = \zeta(m_{y+2}^2|\mathbb{R})$, when $g = y+1$, a contradiction.

Thus, from this we have $dim(M_n) \ge 3$, implying that $dim(M_n) = 3$, $\forall\, n \ge 6$.

Case(II) $n \equiv 1(mod\ 2)$.
We set $n = 2y+1$, $y \in \mathbb{Z}^+$ and $y \ge 3$. Let $U = \{j_2, j_{y+1}, j_n\} \subset V(M_n)$. Next, each vertex of M_n has given metric coordinate corresponding to the taken set U.

For vertices over j-cycle, i.e., $\{j_\alpha : 1 \le \alpha \le n\}$, the metric co-ordinates are

$$\zeta(j_\alpha|U) = \begin{cases} (1, y, \alpha) & \alpha = 1; \\ (\alpha-2, y-\alpha+1, \alpha) & 2 \le \alpha \le y; \\ (\alpha-2, y-\alpha+1, 2y-\alpha+1) & \alpha = y+1; \\ (\alpha-2, \alpha-y-1, 2y-\alpha+1) & \alpha = y+2; \\ (2y-\alpha+3, \alpha-y-1, 2y-\alpha+1) & y+3 \le \alpha \\ & \le 2y+1. \end{cases}$$

For the vertices $\{k_\alpha : 1 \le \alpha \le n\}$, the metric co-ordinates are $\zeta(k_\alpha|U) = \zeta(j_\alpha|U) + (1,1,1)$ for $1 \le \alpha \le n$. Next, for the vertices $\{l_\alpha : 1 \le \alpha \le n\}$, the metric co-ordinates are $\zeta(l_\alpha|U) = \zeta(k_\alpha|U) + (1,1,1)$ for $1 \le \alpha \le n$.

For the vertices $\{m_\alpha : 1 \le \alpha \le n\}$, the metric co-ordinates are

$$\zeta(m_\alpha|U) = \begin{cases} (3, y-\alpha+3, \alpha+3) & \alpha = 1; \\ (\alpha+1, y-\alpha+3, \alpha+3) & 2 \le \alpha \le y; \\ (\alpha+1, \alpha-y+2, 2y-\alpha+3) & \alpha = y+1; \\ (2y-\alpha+5, \alpha-y+2, 2y-\alpha+3) & y+2 \le \alpha \le 2y; \\ (2y-\alpha+5, \alpha-y+2, 3) & \alpha = 2y+1. \end{cases}$$

Finally, for the vertices $\{o_\alpha^s : 1 \le \alpha \le n, 1 \le s \le q\}$, the metric co-ordinates are $\zeta(o_\alpha^s|U) = \zeta(m_\alpha|U) + (s,s,s)$ for $1 \le \alpha \le n$.

Next, these codes for all vertices in M_n are unique and distinct from one and an other in at least one coordinate, which results in $dim(M_n) \le 3$. Assuming that $dim(M_n) = 2$, then as in Case (I), we have the same contradictions. Therefore, we have $dim(M_n) = 3$ as well in this case, which proves the theorem. □

Proposition 3.2. *The resolving independent number for* M_n *is* 3, $\forall\, n \ge 6$.

Proof. For the proof, follow Theorem 3.1. □

4. Conclusion and Discussion

Obtaining resolving set for novel planar structure plays an important role in interconnection networks for the transmission of the data. In this article, we proved that $dim(L_n) = dim(M_n) = 3$, where L_n and M_n are obtained by joining q-copies of the prism graph on R_n [9] and U_n [7], respectively. Further, we demonstrated for these two planar convex polytope graphs that the cardinality of respective resolving independent set is also three for them. Further, several other variations of MD were also introduced in last two decades, such as edge metric dimension, local metric dimension, strong metric dimension, mixed metric dimension, etc [11, 12, 14]. Therefore, in future we will try to investigate several other variants of MD for the planar graphs L_n and M_n.

5. Acknowledgements

We would like to express our sincere gratitude to the referees for their careful reading of this manuscript and for all of their insightful comments/criticism, which have resulted in a number of major improvements to this manuscript..

5.1. Authorship contribution:

M. Gayathri is sole author of this article.

5.2. Funding:

No funding was used in this study.

5.3. Conflict of interest:

No conflicts of interest.

5.4. Declaration:

This research has been conducted ethically, reporting of those involved in this article.

5.5. Similarity Index:

I hereby confirm that there is no similarity index in abstract and conclusion while overall is less than 5% where individual source contribution is 2% or less than it.

5.6. Data Availability

Data sharing is not applicable to this article as no data set were generated or analyzed during the current study.

References

[1] M. A. Lisitsyna and S. V. Avgustinovich, Siberian Electron Math Rep Vol.13, 1116 (2016). https://doi.org/10.17377/semi.2016.13.088

[2] F. Harary and R. A. Melter, On the metric dimension of a graph, Ars Comb Vol.2, 191 (1976).

[3] P. J. Slater, Leaves of trees, Cong Numeran Vol.14, 549 (1975).

[4] G. Chartrand et al., Math Bohem Vol. 128, 379 (2003). http://dx.doi.org/10.21136/MB.2003.134003

[5] S. Khuller et al., Discret Appl Math Vol.70, 217 (1996). https://doi.org/10.1016/0166-218X(95)00106-2

[6] J. Cáceres et al., SIAM J Discrete Math Vol. 21(2), 423 (2007). https://doi.org/10.1137/050641867

[7] M. Imran et al., Comput Math with Appl Vol.60(9), 2629 (2010). https://doi.org/10.1016/j.camwa.2010.08.090

[8] A. Sebo and E. Tannier, Math Oper Res Vol.29(2), 383 (2004). https://doi.org/10.1287/moor.1030.0070

[9] M. A. Malik and M. Sarwar, Afr Math Vol.27, 229 (2016). https://doi.org/10.1007/s13370-015-0336-5

[10] S. K. Sharma and V. K. Bhat, Discrete Math Algorithms Appl Vol.13(1), 2050095 (2021). https://doi.org/10.1142/S1793830920500950

[11] S. K. Sharma et al., Chem Zvesti Vol.76(7), 4115 (2022). https://doi.org/10.1007/s11696-022-02151-x

[12] A. Kelenc et al., Discret Appl Math Vol.31, 204 (2018). https://doi.org/10.1016/j.dam.2018.05.052

[13] S. K. Sharma et al., Front Phys Vol.9, 600 (2021). https://doi.org/10.3389/fphy.2021.749166.

[14] A. Kelenc et al., Appl Math Comput Vol.314, 429 (2017). https://doi.org/10.1016/j.amc.2017.07.027

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